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APPLICATION OF THE HOLLOMON EQUATION TO THE METAL DRAWING GOODNESS DEGREE

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Abstract

In order to establish the cold wire drawing production, the CuNi2Si alloy passes schedule were examined by the Duckfield and Ermanok methods in two wire drawing versions: with the diamond die with an angle of 9^o and the tungsten carbide die with an angle of 7^o. Experiments confirmed that the CuNi2Si alloy can be successfully transformed to the wire with the diameter of 0.2 mm with properties enabling its practical application. The experimental results and the control group showed that the theoretical methods of the upper and lower bound solution can determine the values of the relevant cold wire drawing factors for the CuNi2Si alloy to which Hollomon curve applies.

Keywords: Hollomon's equation, CuNi2Si, lower bound solution, upper bound solution

Introduction

Most polycrystalline aggregates show approximately parabolic relationship between the true stress and true strain. For metals and alloys to which the Hollomon equation applies, the true stress at which plastic instability appears is expressed through equation [1]:

$$\sigma = C\varepsilon_n^n \tag{1}$$

where: σ [Nm^{-2}] – stress
C [Nm^{-2}]– the strengthening factor
n – index of the strain hardening
ε_n- real homogeneous strain
The drawing stress is expressed as:

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$$\sigma_v = \frac{1}{\eta}\int_0^{\varepsilon} C\varepsilon^n d\varepsilon = \frac{C\cdot\varepsilon^{n+1}}{\eta(n+1)} = \frac{\sigma_e}{\eta(n+1)}\varepsilon \tag{2}$$

where η is the goodness degree for the efficient action during drawing [2].
The bound workability is:

$$\varepsilon_{max} = \eta(n+1) \tag{3}$$

and the reduction of the section is calculated from:

$$\varepsilon_{max} = \ln\left(\frac{A_b}{A_a}\right)_{max} \Rightarrow \left(\frac{A_b}{A_a}\right)_{max} = e^{\varepsilon max} \tag{4}$$

$$\left(\frac{A_b}{A_a}\right)_{max} = e^{\eta(n+1)} \tag{5}$$

where: A_b and A_a [m^2] – cross section before and after drawing

The strain rate $\dot{\varepsilon}$ for the round profile during wire drawing is given as [3]:

$$\dot{\varepsilon} = \frac{\ln\lambda}{t_d}\frac{6tg\alpha\cdot v_i\ln\lambda}{\left(D_b - D_a\right)\cdot\left(\lambda+1+\lambda^{1/2}\right)} \tag{6}$$

where: $\dot{\varepsilon}$ [s^{-1}] - strain rate

D_b and D_a [m] – diameters of the drawing material before and after drawing, respectively

v_i [ms^{-1}] – the velocity of the drawing
α [rad] – the die semi-angle
t_d – time derivative
ln λ – is ε, where ε is logarithmic strain

At the lower bound solution (LBS) the optimum die semi-angle for the minimal drawing force is shown as [4, 5]:

$$\alpha_{op} = \sqrt{\frac{K_m\Delta A\mu}{0.77\,A_a K_{fm}}} \tag{7}$$

where $\Delta A = A_b - A_a$

When the curve by S. Geleji is applied to (1) and when $K_m = K = \sigma_e$, then:

$$K_{fm} = K_{ai} \frac{2\varepsilon_b + \varepsilon_a}{2\varepsilon_b + \varepsilon_a(1+n)} \tag{8}$$

where: K_{ai} – reshaped material hardness for the whole strain in passes**:**

K_{fm} – average strain hardness

ε_b – strain in passes i

ε_a – strain in passes (i-1)

At the upper bound solution (UBS) the optimum die semi-angle before and after drawing of the material may be calculated from [6, 7]:

$$\alpha_{op} = \left(\frac{2}{3}\sqrt{3}\mu\left(1 + \ln\frac{R_b}{R_a}\right)\ln\frac{R_b}{R_a}\right)^{\frac{1}{2}} \tag{9}$$

where: R_b and R_a [m] – radius of the drawing material before and after drawing, respectively

μ – the friction coefficient from the condition of plastic flow by von Mises' stress is given as:

$$\mu = \frac{m}{\sqrt{3}}$$

where m is the coefficient of the shear.

The stress during wire drawing is given with the following expression [8]:

$$\sigma_{xf} = \sigma_{xb} + \sigma_e \, 2f(\alpha)\ln\left(\frac{R_b}{R_a}\right) + \frac{2}{\sqrt{3}}\sigma_e \left[\left(\frac{\alpha}{\sin^2\alpha}\right) - \mathrm{ctg}\,\alpha + m \cdot \mathrm{ctg}\,\alpha \cdot \ln\left(\frac{R_b}{R_a}\right) + m\left(\frac{L}{R_a}\right)\right] \tag{10}$$

The stress against the tension, under the experimental conditions, is $\sigma_{xb}=0$.

The specific strain work, by Duckfield method, for all passes was calculated from the equation (11) [20]:

$$w_T = 1.12\bar{S}(\varepsilon_T + 0.076n) \tag{11}$$

for the n-th passes was:

$$w_n = -1.12\bar{S}(\varepsilon_T + 0.076n) \tag{12}$$

$\bar{S}$ – average stress strength for the whole deformation (for all passes)

ε_T – logarithmic strain by the pass

The effect of wire drawing passes schedule on the multiple-drawing machines was the subject of interest in many investigations [8-19]. These investigations were based on the experimental data, such as, passes schedule with: the constant reduction, a decreasing single reduction, a decreasing logarithmic strain, as well as Duckfield and Ermanok methods [20, 3]. Using the theoretical methods for analyzing the passes schedule calculation it was possible to predict the passes schedule accuracy from the LBS and UBS results for the metals to which the Hollomon curve applies.

The objective of this paper was to study metal drawing goodness degree for CuNi2Si alloy to which the Hollomon equation applies. For the analysis of strain characteristics of this alloy, the stress-strain curves were used, where the true stress σ_e is shown in the relationship to the true or the logarithmic strain ε.

The experimental investigations were done on the basis of the experimental plan factors in several levels:

- the investigation on the die geometry, in particular the cone-convergent die part
- the strain hardening curve determination necessary for the comparative method evaluation for the passes schedule calculation
- the friction coefficient determination between the die material and the drawing material
- the effective stress curve values determination in the form of $K = C\varepsilon^n$ for the material used the plan on the introduction of mathematical methods for the investigation of the stress-strain state by the LBS and UBS in order to comparatively evaluate passes schedule methods

Experimental

The semi manufactured CuNi2Si alloy in the form of wire with the chemical composition (weight %): Ni (1.6 – 2.5), Si (0.5 – 0.8), impurities (max. 0.5) and Cu being remainder, was the object of this investigation. Preparation of the starting wire was done on the industrial equipment at the laboratory of the Cable factory Novkabel, Novi Sad. The starting diameter of wire was D_o = 1.288 mm. The alloy was homogenized by annealing in the nitrogen atmosphere at 900°C for 30 min and water quenched. The final wire drawing from D_o = 1.288 mm to D_n = 0.200 mm was performed at the Faculty of Technology and Metallurgy, Belgrade. The drawing was executed on the laboratory drawing bench for the measurement of the drawing forces and on the rotating wire drawing machine. The hypoid oil was used as a lubricant. The diamond and the tungsten carbide dies were used, according to DIN 1546 and DIN 1547 [21, 22]. Length of the cylindrical die part was L=0.2 D_i. A micrometer with an accuracy of 1/1000 mm was used for measurements of wire diameter. The tensile tests were performed at the laboratory of the Copper Mill Sevojno on the testing machine Zwick 1484.

Results

For the passes schedule calculation, by Duckfield and Ermanok methods, the stress-logarithmic strain curves were constructed for the CuNi2Si alloy (Fig. 1. and 2.).

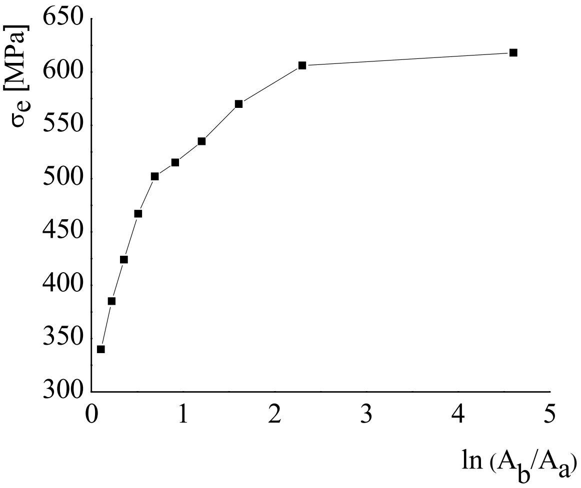

Fig. 1. The dependence of the true stres σ_e [MPa] on the strain by the Duckfield method for the CuNi2Si alloy

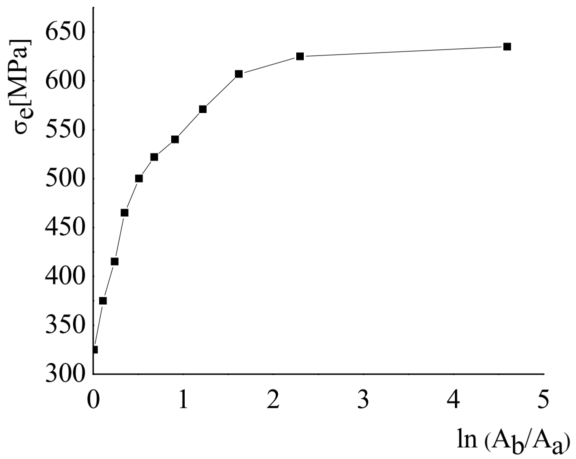

Fig. 2. The dependence of the true stress σ_e [MPa] on the strain by the Ermanok method for the CuNi2Si alloy

Strain hardening curve was determined experimentally (Fig. 3.) and was calculated using the regression analysis (13):

$$K = 528.9\varepsilon^{0.24} \text{ [MPa]} \tag{13}$$

The experiment was based on the planned passes schedules: A, B, C and D. The variant A comprised 20 passes with the standard diamond die, with an angle of 9°, with the decreasing logarithmic strain with the mean value of ε_{sr} =0.184 and the whole strain was ε_u =3.725. The variant B, was the same as A, just the tungsten carbide die was used instead of the diamond die, with an angle of 7°. The variant C comprised 13 passes with the standard diamond die, with an angle of 9°, with the constant logarithmic strain ε_{sr} =0.286 and the whole strain was ε_u =3.725. The variant D, was the same as C, just tungsten carbide die was used instead of the diamond die, with an angle of 7°.

It was expected that the passes schedules would provide approximately the same deformation work of $Q_0 = 158 MJ \cdot m^{-3}$, calculated by the equation (12). The strain rate was calculated by the equation (6). The drawing rate was $v_i = 58.4 mm \cdot s^{-1}$. The tensile strength was obtained from the curves in Fig. 1 and 2. Then, the passes schedules were assembled under real conditions and the experimental values of the following elements were determined: drawing force, drawing stress, effective stress, relative drawing stress and the total specific strain work. The friction coefficient determination, for all passes schedule variants, was determined by a separate experiment and the results are depicted in Table 1. The relative drawing stress was calculated from the equation (14):

$$\upsilon = \frac{\sigma_{xf}}{\sigma_e} \qquad (14)$$

and is depicted in Fig. 4 and 5. All other results obtained are depicted in Table 1.

Table 1. Total and average strain work and friction coefficient for the passes schedule A, B, C and D

Plan	Total strain work, w_u [MPa m^{-3}]	Average strain work, w_{sr} [MPa m^{-3}]	Friction coefficient, μ
A	4986	262	0.060
B	8072	403	0.272
C	4767	368	0.061
D	5443	418	0.165

Effective stress curves, for all four variants, and the Hollomon curve are depicted in Fig. 3. The experimental results analysis indicated that all curves correspond to the third stage of strain hardening [23]. The curves differ from each other due to two reasons. Due to the strain hardening process whose speed increases with increasing the deformation disparities due to the drawing angle increase, partial deformation and friction coefficient. The other reason is due to the recovery process which reduces the strain hardening rate, since its intensity is proportional to the total brought energy and inversely proportional to the strain rate.

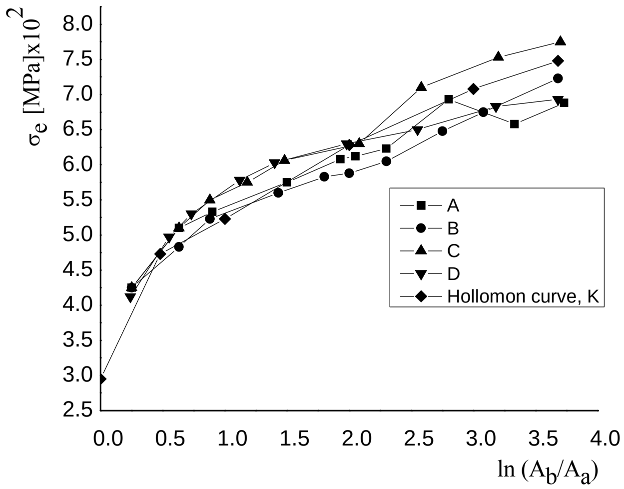

Fig. 3. The relation ln (A_o/A_i) versus stress σ_e for the A – D passes schedules and the Hollomon curve K

It should be expected, due to the strain hardening process, that there is the following order: $\sigma_{Ce} > \sigma_{De} > \sigma_{Ae} > \sigma_{Be}$, while the brought specific works values should be in order: $w_B > w_D > w_A > w_C$. The partial strain work w_i was two times higher for the passes schedule A and B, while for the passes schedule C and D was 1.8 times higher. The significance of this analysis is that the strain hardening curve approximates well for all the curves of the cold drawing process. The experimentally obtained data were used in the relation (10) to calculate the friction coefficient for each passes schedule. Using the Hollomon curve it was possible to prove a connection between the logarithmic strain and the index of the strain hardening. From the equation (2), the strain hardening speed is: $\frac{d\sigma}{d\varepsilon} = Cn\varepsilon_n^{n-1}$. Taking into account, also, the condition for the occurrence of plastic instability, which is defined as: $\frac{d\sigma}{d\varepsilon} = \sigma$ and at the moment at which plastic instability appears the following equation can be written:

$$nC\varepsilon_n^{n-1} = C\varepsilon_n^n \Rightarrow \varepsilon_n = n \qquad (15)$$

According to the equation (15) the instability occurs when the real homogeneous elongation becomes equal to the strain hardening index. Practically, the theoretical approach for the analysis of the thin wire drawing technological process can be carried out using the previous experiment as a control group. For the comparative evaluation of the single method calculation it was necessery to find out the etalon "series"- the passes schedule.

As there was a search for a standard die angle which was expected to become the optimal one, for this analysis, the equations (9) and (7) were used. When the quadratic equation (9), for the optimal angle, is written in the form:

$$\mu\varepsilon^2 + 2\mu\varepsilon - 2\sqrt{3}\bar{\alpha}_{op}^2 = 0 \qquad (16)$$

the optimum interdependences of its factors can be calculated. In the equation (16) $\bar{\alpha} = f(\mu, \varepsilon)$, where ε is the independent variable and the μ is a dependent variable. When tg $\alpha=\mu$, only then and only then the equation (16) has optimal solutions. For the standard angles the equation (16) can be used, with an error of less than 1%. Then $tg\alpha = \bar{\alpha}$ and the calculation gives the following values for the independent variable ε:

for 7^o ε=0.192
for 9^o ε=0.240

When the equation (4) is written in a form:

$$D_i = D_{i-1} \cdot \exp^{-\frac{\varepsilon}{2}} \qquad (17)$$

Then, according to the equation (17), the etalon series values were calculated. Thus, for the angle of 7^o obtained was the final diameter D_n = 0.200 mm, the number of passes was z = 19. For the angle 9^o, z = 17 and the inlet diameter was D_0 = 1.288 mm. Practical work has confirmed that this passes schedule calculation is valid for the tungsten carbide and diamond dies. But, for the relevant values calculation of the technological drawing process the procedure is different for tungsten carbide die in comparison with the diamond die. So, for the tungsten carbide dies the elements of the system can be obtained from the equation (9) and the Hollomon curve. For the diamond die the elements of the system can be obtained from the equation (7) and the Hollomon curve.

This applies to passes schedules under real conditions and for the etalon series passes schedule. In this paper, the investigation was focused on passes schedule used to organize the CuNi2Si thin wire production. For the ideal series the relevant parameters were calculated with the constant logarithmic strain, which were:

ν - the relative drawing stress from the equation (13)
η – goodness degree from the equation (2)
σ_{xf} - drawing stress from the equation (10)
σ_e - effective stress from the Hollomon equation

The values for the ν and η are constant for ideal passes schedules. For tungsten carbide die with: an angle of 7^o and the logarithmic strain ε = 0.192, the goodness degree was η = 0.295 and the relative drawing stress was ν = 0.678. For the diamond die with: an angle of 9^o and the logarithmic strain ε = 0.240, the goodness degree was η = 0.380, and the relative drawing stress was ν = 0.512.

The values for the passes schedule A and B are depicted in the Fig. 4 and for the C and D in the Fig. 5, calculated by the same procedure.

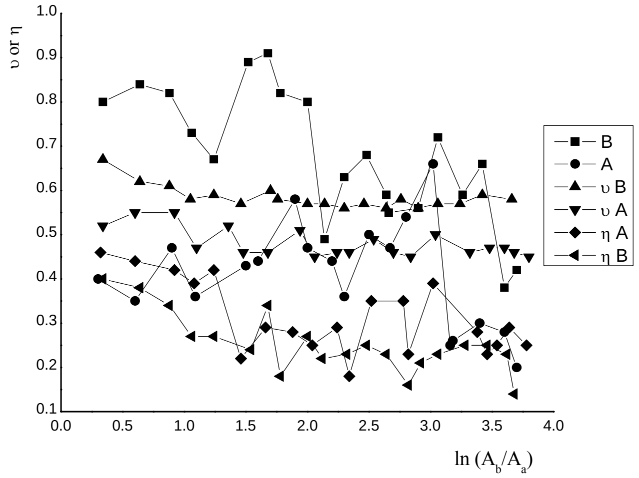

Fig. 4. The arrangement of the relative drawing stress σ_{xf}/σ_e for the processes A and B, the experimental values. The arrangement of the relative drawing stress σ_{xf}/σ_e for the processes A and B, calculated values, υA and υB The goodness degree η for the A and B, calculated values, ηA and ηB

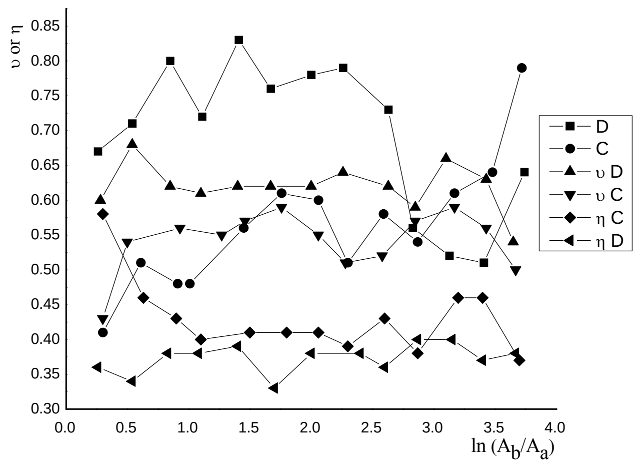

Fig. 5 The arrangement of the relative drawing stress σ_{xf}/σ_e for the processes C and D, the experimental values. The arrangement of the relative drawing stress σ_{xf}/σ_e for the processes C and D, calculated values, υC and υD. The goodness degree η for the C and D, calculated values, ηC and ηD

Discussion

The theory is valid only when confirmed in practice. In this sense, it was necessary to check the correctness of the equation (9). This can be done through determining the friction coefficient values from the experimental data according to the equation (10). The control group used was the passes schedule D. The friction coefficient mean value calculated for the whole passes schedule was, with a slight difference equal to tg α_{ST}. From the experimental results calculated was μ_{sr} = 0.12383. While from the experimental results and Hollomon equation obtained was μ_{sr} = 0.121428. The same procedure was used in case of the equation (7). Compared were the friction coefficient values calculated for the passes schedule with the constant logarithmic strain C. The results obtained for the friction coefficient from the equation (7) and (10), for the passes schedule C, differ slightly. How the series A with the logarithmic decreasing strain in one of the passes has the value n = ε_{op}, which from the equation (10) gives a friction coefficient value of μ = 0.0633, while the friction coefficient, calculated by the equation (7), was μ = 0.0648 showed that the difference is minor. This, also, verified the correctness of the method used. From the presented arises that the certain method application is determined by the die material: for the tungsten carbide die the equation (9) is used, and for the diamond dies the equation (7) is used. The explanation, why this is so, can be sought from the process deriving the most favorable angles. One angle arises from the minimization of the drawing angle (9), the other angle arises from the minimization of the drawing force (7) [5, 6].

For the analysis of the condition in the convergent cone die section necessary was to observe two basic values: friction coefficient and logarithmic strain with the assumption that the die angle is standard. When the standard die angle is the optimal, that can be determined from the logarithmic strain and the optimal die angle can be higher or smaller from the perfect one. In case when ε is higher than the perfect value, then the drawing stress values σ_{xf} increase, the relative drawing stress ν increases, while the goodness degree value η decreases. Valid is, also, the vice versa case. In other words, when the stress is higher than the optimal in the die conical part the contact surface and the material volume increases. Therefore, the drawing force increases, more precisely the drawing stress σ_{xf}. After gathering the previous results, the relevant values of the technological drawing metal process can be analyzed. Thus, the curve A slightly deviates from the curve νA. Similarly, the curve C slightly deviates from the curve νC. The deviation of the curves B from the curve νB, as well as the curve D from the curve νD is significant. The goodness degree curves η can be analyzed by the same procedure from the Figure 5. Significant deviations are the result of higher real logarithmic strain values compared to the ideal passes schedules. The previous findings indicate that in the practice passes schedules with constant logarithmic strain should be used, and in the passes schedule preparation in practice, there should be only small passes deviations from ideal ones.

Conclusion

Using CuNi2Si alloy for the thin wire drawing it was established that the product has good mechanical properties. The theoretical methods: the lower bound solution (LBS) and the upper bound solution (UBS) can be used for the calculation and analysis of the technological drawing processes, but only for metals to which the Hollomon equation applies.

It can be concluded that the standard die angles were accurately choosen. The Duckfield method for the passes schedule calculation is based on the original idea which predicts in advance the system quality. The advantage of this method is that it is based on the constant logaritmic strain and, in this form, it can be recommended for the practical application.

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