Association of Metallurgical Engineers of Serbia
AMES

Scientific paper
UDC: 678:534.21

DYNAMIC CHARACTERISATION OF COMPOSITE LATTICE PLATES BASED ON FSDT

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Received 15.09.2010
Accepted 06.10.2010

Abstract

A new useful approach, based on the first order shear deformation theory (FSDT) is presented for the free vibration analysis of the simply supported composite lattice plates (CLP). Unlike the previous theory, the in-plane normal and shear stresses are considered. The governing equations and the boundary conditions are derived by Hamilton's principle. The closed form solution was carried out using the Navier method in order to solve eigenvalues. The effects of non-dimensional natural frequency with respect to the various parameters of composite lattice plates are shown in this paper.
Key words: Composite Lattice Plate, Free Vibration, FSDT, Lattice Stiffness Matrix

Introduction

Nowadays, composite lattice structures have low weight and high special strength, and have found applications in aerospace industries. Composite lattice plates (CLP) play a big role in aerospace industries in fuselage, airplane wing, helicopter blade trail and launch vehicle applications. This has resulted in an extensive research work in the field of lattice structures [1-8]. Among many researches of lattice structures, Vasiliev *et al.* [9-11] have presented an integrated design, manufacturing and testing process for high performance lattice structures made by continuous filament and wet winding processes from carbon and armed epoxy composites which used as structural elements of airplane frames and space launch vehicles. In their papers, lattice structures with regular and dense system of ribs are simulated by continuum models, in order to obtain their stiffness matrix. Composite lattice plates are widely used in engineering fields. These structures are subjected to external dynamic loads which can cause the undesirable resonance causing fatigue. Thus, the provocation of each natural frequency should be prevented in the structures as much as possible. Therefore, knowing the natural frequency values and the vibration mode types of the structures seems to be

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necessary. Chen and Tsai research gave an analytical model for modal analysis of composite lattice plates [12]. This model, so called ESM (Equivalent Stiffness Model), allows the modal analysis using stiffness matrix of lattice plate. However there are a few published papers about free vibration analysis of composite lattice plates. Comprehensive review about vibration analysis of lattice plates and shells are presented by G. I. Pshenichnov [13]. The classical and the first-order shear deformation theories are among the most useful theories for composite lattice structures analysis [14-16]. The classical theory does not afford enough accuracy for analysis of thick plate, because of ignoring the transverse shear deformations and overestimating natural frequencies. Therefore, calculating the effect of transverse shear deformations in order to investigate the thick plate seems to be necessary. Calculation accuracy mostly depends on shear correction coefficient in first-order shear theory [17, 18]. Considering the pattern of lattice structures, the first order theory is needed for accurate calculation of natural frequencies of the lattice plate.

In this paper, the free vibration analysis of simply supported composite lattice plate based on first-order shear deformation theory (FSDT) is presented. The in-plane normal and shear stresses are considered.

Governing equations and analytical solving method

The studied composite lattice plate in this paper is rectangular. A composite lattice plate with its variables is represented in Fig. 1.

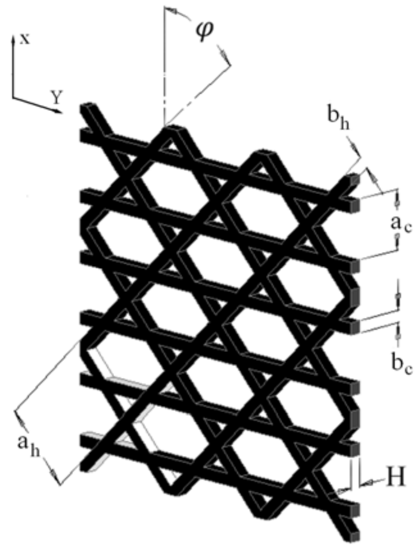

Fig 1. Geometrical parameters of a composite lattice plate

These variables include:

H = Height of cross-section of ribs (thickness of lattice plane)

b_h = Thickness of cross-section of helical ribs

b_c = Thickness of cross-section of horizontal rib

a_h = Distance between helical ribs

a_c = Distance between horizontal ribs

φ = Angle of helical ribs with axis (x)

Movement equations and boundary conditions are achieved by Hamilton's principle.

$$\delta\int_{t_1}^{t_2}(U+V-T)\,dt=0 \tag{1}$$

Where *U, V* and *T* are the potential energy, the kinetic energy and the external forces derived energy, respectively. δ is the operator of the first variation. t_2-t_1 is the time interval. The variation equation of potential energy is presented in the following. In the Eq. (2), σ_{ii} is normal stresses in *x* and *y* directions. τ_{iz} and γ_{iz} (*i*=*x* or *y*) are normal shear stresses and strains, respectively. V is the volume of the lattice plate.

In this analysis the terms of in-plane energy variations are regarded in Eq. (2). Those terms of in-plane energy changes which are not considered in previous study are regarded in Eq. (2) too.

$$\delta U=\int_V\left(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{yy}\delta\varepsilon_{yy}+\tau_{xy}\delta\gamma_{xy}+\tau_{xz}\delta\gamma_{xz}+\tau_{yz}\delta\gamma_{yz}\right)dv \tag{2}$$

The first-order shear theory is used for the lattice plate considering small displacements and rotations.

$$u(x,y,z,t)=u_0(x,y,z,t)+z\psi_x(x,y,z,t)$$
$$v(x,y,z,t)=v_0(x,y,z,t)+z\psi_y(x,y,z,t)$$
$$w(x,y,z,t)=w_0(x,y,z,t) \tag{3}$$

The stress and density resultants equal:

$$\{N_{xx},M_{xx}\}=\int_{-h/2}^{h/2}(1,z)\sigma_{xx}dz$$
$$\{N_{yy},M_{yy}\}=\int_{-h/2}^{h/2}(1,z)\sigma_{yy}dz$$
$$\{N_{xy},M_{xy}\}=\int_{-h/2}^{h/2}(1,z)\tau_{xy}dz$$
$$Q_{xz}=\int_{-h/2}^{h/2}\tau_{xz}dz$$
$$Q_{yz}=\int_{-h/2}^{h/2}\tau_{yz}dz$$
$$I_n=\int_{-h/2}^{h/2}(z^n\rho)dz \tag{4}$$

Equations of equilibrium in terms of displacement requires calculation stiffness matrix of lattice structure. Hence it is necessary to calculate [A], [B] and [D] matrixes components in terms of lattice plate geometric parameter. According to the equations for the [A], [B] and [D] matrixes of orthotropic panel, one of the newest and most simple way to achieve [A], [B] and [D] matrixes of lattice plate, is the use of [Q] matrix.

According to geometrical variable of lattice plates, constants of $([Q]_{lattice})_{xy}$ matrix are:

$$\begin{bmatrix} \frac{2E_h b_h}{a_h c^4} & \frac{2E_h b_h}{a_h s^2 c^2} & 0 \\ Q_{21}=Q_{12} & \frac{2E_h b_h}{a_h s^4}\frac{E_c b_c}{a_c} & 0 \\ 0 & 0 & \frac{2E_h b_h}{a_h s^2 c^2} \end{bmatrix} \qquad s=\sin\phi \text{ , } c=\cos\phi \tag{5}$$

This matrix is the stiffness matrix of a lattice plate in (x) and (y) general coordination as shown in Fig. 1. This matrix is similar with the matrix which is obtained by Tsai [12] and Vasiliev [9-10] (it is noteworthy that in Tsai's research, the obtained matrix is computed considering the vertical ribs which is not used in the present study).

[A], [B] and [D] matrices for lattice plates, are now computable by equations used for calculation of these matrices. Stiffness matrix of a lattice plate is presented as below:

$$[A]=\int_{-H/2}^{H/2}\left([Q]_{lattice}\right)_{xy}dz=H\left([Q]_{lattice}\right)_{xy} \tag{6}$$

$$[B]=\int_{-H/2}^{H/2}\left([Q]_{lattice}\right)_{xy}zdz=0 \tag{7}$$

$$[D]=\int_{-H/2}^{H/2}\left([Q]_{lattice}\right)_{xy}z^2dz=1/12H^3\left([Q]_{lattice}\right)_{xy} \tag{8}$$

With achieved relations for [A], [B] and [D] matrices the relation of force and applied moments with strains and curvatures of lattice plates can be obtained.

Using Eqs. (1-8), the first variation of the kinetic energy and the variation of the external forces according to the reference [19] and also performing mathematical operations, gives the system of governing equations of motion which is expressed as follows:

$$\begin{aligned} &A_{11}u_{0,xx}+A_{12}v_{0,xy}+A_{16}(u_{0,xy}+v_{0,xx})+B_{11}\psi_{x,xx}+B_{12}\psi_{y,xy} \\ &+B_{16}(\psi_{x,xy}+\psi_{y,xx})+A_{16}u_{0,xy}+A_{26}v_{0,yy}+A_{66}(u_{0,yy}+v_{0,xy}) \\ &+B_{16}\psi_{x,xy}+B_{26}\psi_{y,yy}+B_{66}(\psi_{x,yy}+\psi_{y,xy})+\overline{n}_x-I_0u_{0,tt}-I_1\psi_{x,tt}=0 \end{aligned} \tag{9}$$

$$B_{11}u_{0,xx}+B_{12}v_{0,xy}+B_{16}(u_{0,xy}+v_{0,xx})+D_{11}\psi_{x,xx}+D_{12}\psi_{y,xy}$$
$$+D_{16}(\psi_{x,xy}+\psi_{y,xx})+B_{16}u_{0,xy}+B_{26}v_{0,yy}+B_{66}(u_{0,yy}+v_{0,xy})$$
$$+D_{16}\psi_{x,xy}+D_{26}\psi_{y,yy}+D_{66}(\psi_{x,yy}+\psi_{y,xy})-A_{55}(\psi_{x}+w_{0,x})$$
$$-A_{45}(\psi_{y}+w_{0,y})-I_{1}u_{0,tt}-I_{2}\psi_{x,tt}=0 \tag{10}$$

$$A_{55}(\psi_{x,x}+w_{0,xx})+A_{45}(\psi_{y,x}+w_{0,xy})+A_{45}(\psi_{x,y}+w_{0,xy})$$
$$+A_{44}(\psi_{y,y}+w_{0,yy})+q_{t}-I_{0}w_{0,tt}=0 \tag{11}$$

$$A_{12}u_{0,xy}+A_{22}v_{0,yy}+A_{26}(u_{0,yy}+v_{0,xy})+A_{16}u_{0,xx}+A_{26}v_{0,xy}$$
$$+A_{66}(u_{0,xy}+v_{0,xx})+B_{12}\psi_{x,xy}+B_{22}\psi_{y,yy}+B_{26}(\psi_{x,yy}+\psi_{y,xy})$$
$$+B_{16}\psi_{x,xx}+B_{26}\psi_{y,xy}+B_{66}(\psi_{x,xy}+\psi_{y,xx})+\overline{n}_{y}-I_{0}v_{0,tt}-I_{1}\psi_{y,tt}=0 \tag{12}$$

$$B_{12}u_{0,xy}+B_{22}v_{0,yy}+B_{26}(u_{0,yy}+v_{0,xy})+B_{16}u_{0,xx}+B_{26}v_{0,xy}$$
$$+B_{66}(u_{0,xy}+v_{0,xx})+D_{12}\psi_{x,xy}+D_{22}\psi_{y,yy}+D_{26}(\psi_{x,yy}+\psi_{y,xy})$$
$$+D_{16}\psi_{x,xx}+D_{26}\psi_{y,xy}+D_{66}(\psi_{x,xy}+\psi_{y,xx})-A_{45}(\psi_{x}+w_{0,x})$$
$$-A_{44}(\psi_{y}+w_{0,y})-I_{1}v_{0,tt}-I_{2}\psi_{y,tt}=0 \tag{13}$$

Equations system consists of 5 equations and 5 unidentified coefficients which is the minimum number of independent equations and is applicable for all conditions, especially for various boundary conditions. The 5 dependent unidentified coefficients are:

$$\{u_0,\ \psi_x,\ v_0,\ \psi_y,\ w_0\} \tag{14}$$

It is shown that the differential equations of free vibrations of a lattice plate versus in-plane, transverse displacements and rotations functions can be transformed to a standard problem of Eigen values by the use of Navier method. This method reduces the equations of a composite lattice plate with simply supported boundary conditions to the following Eigen value equation:

$$\left([K]-[M]\omega^2\right)\left\{X_0^*\right\}=\{0\} \tag{15}$$

In Eq. (15), [*K*] and [*M*] are the stiffness and mass matrices, respectively. The terms of each [*K*] and [*M*] matrices are shown as below:

$$K_{(1,1)}=A_{(1,1)}\alpha_m^2+A_{(6,6)}\beta_n^2 \qquad K_{(1,2)}=(A_{(1,2)}+A_{(6,6)})\alpha_m\beta_n$$
$$K_{(2,1)}=K_{(1,2)} \qquad K_{(2,2)}=A_{(6,6)}\alpha_m^2+A_{(2,2)}\beta_n^2$$
$$K_{(3,3)}=A_{(5,5)}\alpha_m^2+A_{(4,4)}\beta_n^2 \qquad K_{(3,4)}=A_{(5,5)}\alpha_m$$

$$K_{(3,5)} = A_{(4,4)}\beta_n \qquad K_{(4,3)} = K_{(3,4)}$$

$$K_{(4,4)} = D_{(1,1)}\alpha_m^2 + D_{(6,6)}\beta_n^2 + A_{(5,5)} \qquad K_{(4,5)} = (D_{(1,2)} + D_{(6,6)})\alpha_m\beta_n$$

$$K_{(5,3)} = K_{(3,5)} \qquad K_{(5,4)} = K_{(4,5)} \qquad K_{(5,5)} = D_{(6,6)}\alpha_m^2 + D_{(2,2)}\beta_n^2 + A_{(4,4)}$$

$$M_{(1,1)} = M_{(2,2)} = M_{(3,3)} = I_0 \qquad M_{(4,4)} = M_{(5,5)} = I_2 \tag{16}$$

in Eq. (12), $\alpha_m = m\pi / a$ and $\beta_n = n\pi / b$.

Numerical Results and Discussion

In this section an example of lattice plate is investigated. The achieved results of solving the problem with the use of the present theory are compared to the results of the classical plate theory (CPT) and finite element method (FEM).

Numerical Verification

Using the present method, numerical results are obtained for the three considered model of composite lattice plate with geometric and mechanical parameters given in Tables 1 and 2.

Table 1 The geometric parameters of the lattice plate [m]

Example	H	φ	$b_h = b_c = b$	a_h	$a_c = a_h/(2\sin\varphi)$
1	0.02	45°	0.01	0.0707	0.05
2	0.05	45°	0.01	0.0707	0.05

Table 2 Mechanical properties of the lattice plate (Material Glass/Epoxy)

Longitudinal modulus	Transverse modulus	Shear modulus	Poisson's ratio
$E_{rib} = E_1\ (GPa)$	$E_2\ (GPa)$	$G_{12}\ (GPa)$	ν_{12}
60	13	3.4	0.3

The obtained results are compared to the solution of the same problem when the considered structures are modeled with a finite element analysis package ANSYS. The one of mesh generated using this package is shown in Fig. 2.

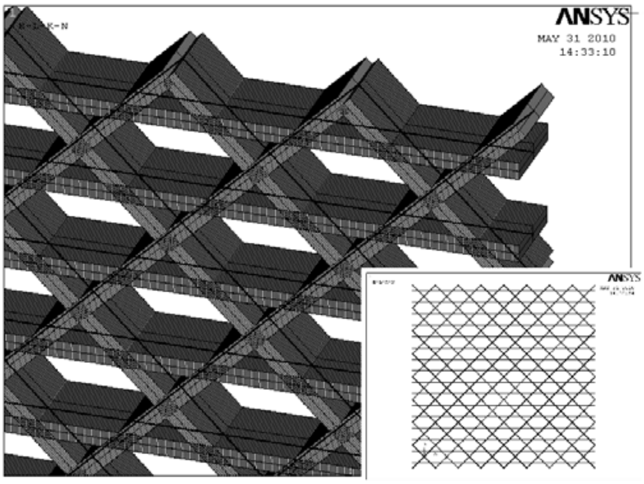

Fig. 2 The mesh generated using ANSYS package for the composite lattice plate (Example 2)

As presented in Table 3, assuming a higher freedom degree in present method and therefore reducing plate stiffness compared to CPT, achieved results of the present method generally show a smaller value compared to CPT. Also, the discrepancies in the results obtained for the present method and FEM are given in Table 3. The observed maximum discrepancy in the present method and FEM results is in the range of 0.002-8.314%. Therefore, the present method is in good agreement with the FEM results compared with CPT results. In addition, the present method is capable of calculating in-plane modes.

Table 3 Compression of values of the natural frequency for considered lattice plates

H	Mode	FEM	CPT	Present	Discrepancy with CPT (%)	Discrepancy with FEM (%)
0.02	1	95.366	95.382	94.468	0.958	0.942
	2	190.73	191.895	189.005	1.506	0.904
	3	265.10	269.201	261.013	3.041	1.542
	4	335.52	341.526	334.118	2.169	0.418
	5	370.57	381.529	367.570	3.659	0.810
0.05	1	225.71	238.456	225.265	5.532	0.197
	2	425.48	479.737	440.073	8.268	3.430
	3	591.83	673.002	570.295	15.261	3.639
	4	700.19	853.816	758.402	11.175	8.314
	5	786.36	953.823	786.343	17.559	0.002

Parametric Study

In this section, the parametric study of example 1 is presented. First and second mode shapes of vertical displacement (*W*) is shown in Fig. 3.

The effect of variation of (h/a) on the lattice plate natural frequencies is shown in Fig. 4.

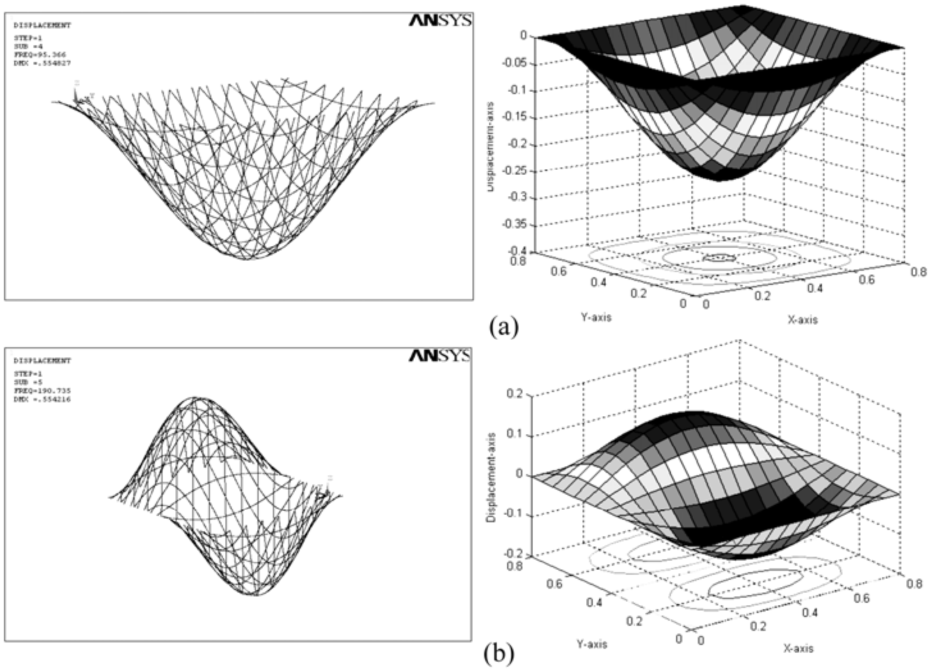

Fig. 3 The three-dimensional view to first and second mode shapes of vertical displacement (W) respectively: (a) and (b), in the composite lattice plate

As can be seen in Fig. 4 increasing the ratio of thickness to length of lattice plate (h/a), the stiffness of structure is increased and thus the value of lattice plate frequency will increase. Fig. 4b shows, when the thicknesses to length ratio are higher than 10%, the discrepancy between the result from the classical method and that obtained from the present method is increased. This point indicates weakness of the classical method in calculating natural frequency of thick lattice plate. The effect of variation (b_c/a_c) on the first three frequencies of structure is shown in Fig. 5. Regarding to this figure, it is obvious that by decreasing rib width and increasing horizontal rib distance, the natural frequency of the lattice plate decreases.

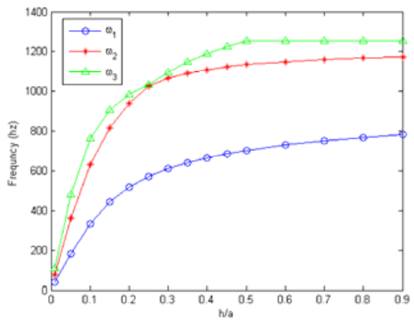

Fig. 4a Variation of natural frequencies versus (h/a) in the composite lattice plate (a) first three natural frequencies

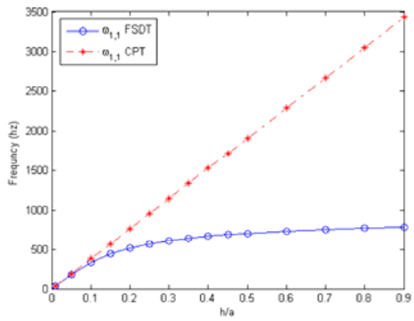

Fig. 4b Variation of natural frequencies versus (h/a) in the composite lattice plate (b) discrepancy between CPT and the present method

According to Fig. 5b, by increasing the ratio (b_c/a_c), discrepancy between the result obtained applying classical method and the present method is increased.

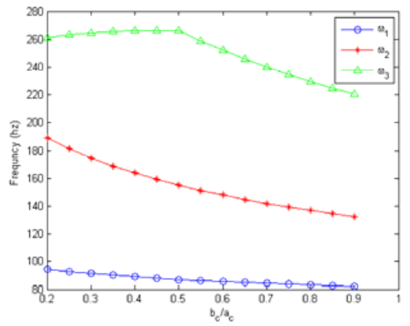

Fig. 5a Variation of natural frequencies versus (b_c/a_c) in the composite lattice plate (a) first three natural frequencies

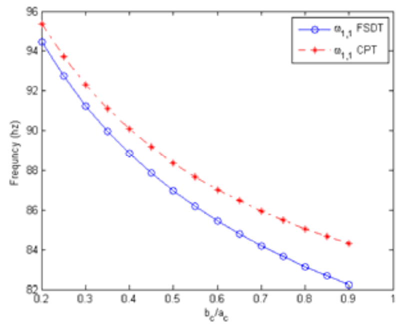

Fig. 5b Variation of natural frequencies versus (b_c/a_c) in the composite lattice plate (b) discrepancy between CPT and the present method

The effect of variation (b_h/a_h) on the first three frequencies of structure is shown in Fig. 6a. The results of this figure show that by decreasing the width and increasing helical rib distance the natural frequency of the lattice plate is increased. According to Fig. 6b by increasing the ratio (b_h/a_h), discrepancy between the resulted obtained from the classical method and the present method is increased.

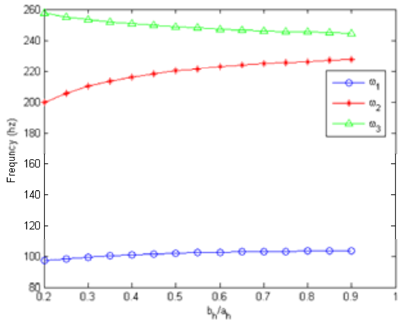

Fig. 6a Variation of natural frequencies versus (b_h/a_h) in the composite lattice plate (a) first three natural frequencies

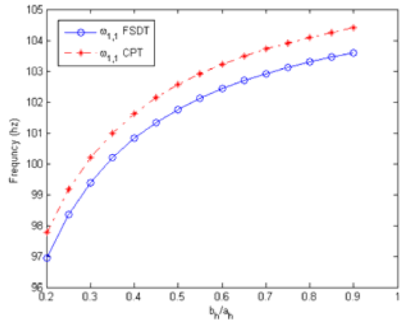

Fig. 6b Variation of natural frequencies versus (b_h/a_h) in the composite lattice plate (b) discrepancy between CPT and the present method

The variation of the first three natural frequencies of the lattice plate according to the helical Rib's angle is shown in Fig. 7.

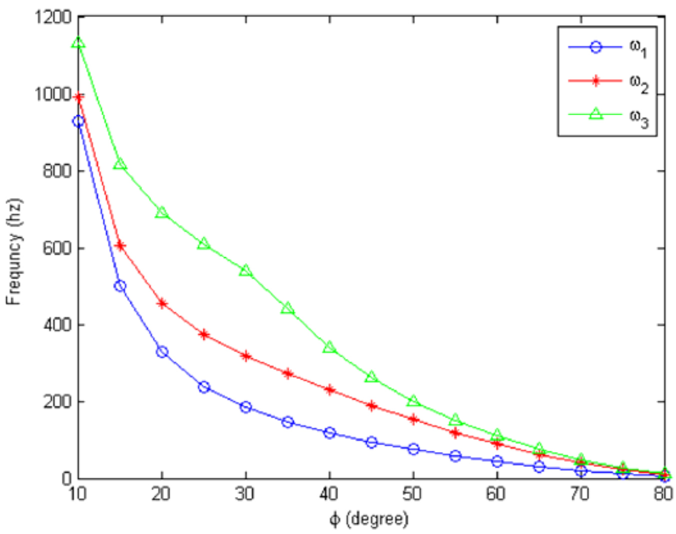

Fig. 7 Variation of natural frequencies versus φ in the composite lattice plate

This figure shows that by increasing φ the natural frequency of lattice plate are decreased. This phenomenon indicates that the lattice plate pattern is changed to orthogrid, so the natural frequency of the lattice plate is decreased. This means that the values of natural frequencies of orthogrid lattice plate are less than other lattice plate patterns.

Conclusion

Free vibration of lattice plates is investigated by using the new formulation. The applied degrees of freedom in the present method are higher than the classical theory, so its stiffness is lower than classical theory and the achieved natural frequencies are closer to FEM values. According to achieved results, it can be seen that thick lattice plate dynamic response significantly depends on the assumption of considering the in-plane stresses in the plate. Difference between the results of the present theory and the classical theory shows this point. For example, natural frequency values and also displacement of the lattice plate are affected by this point. Thus, if not considered in calculations, the in-plane stresses cause the increase of the discrepancy between the classical and first order results.

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